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Influence of Point-like Disorder on the Hall Resistivity Behavior in an Anisotropic Planar Pinning Potential

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Explicit current- and temperature-dependent expressions for anisotropic longitudinal and transverse nonlinear magnetoresistivities are derived and analyzed on the basis of a Fokker-Planck approach for two-dimensional vortex dynamics in a washboard pinning potential in the presence of point-like disorder. Gradually increasing the strength of the point-like pinning (in an experiment this is simply done by irradiation of the sample with different doses of high-energy electrons) this theory predicts a gradual decrease of the anisotropy of the magnetoresistivities. The physics of the transition from the recently discussed new scaling relations for anisotropic Hall resistivities in the absence of point-like pins to the well-known scaling relations for isotropic pinning is elucidated. This is discussed in terms of a gradual isotropization of the guided vortex motion responsible for the existence in a washboard pinning potential of new anisotropic Hall voltages which are odd with respect to magnetic field reversal.

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1. INTRODUCTION

The importance of flux-line pinning in preserving the superconductivity in a magnetic field has been generally recognized since the discovery of type-II superconductivity. But till now the mechanism of flux-line pinning and creep in the high-T_c superconductors (HTSC's) is still a matter of controversy and great current interest[1–6], especially in the cases of strong competition between different types of pins. The main defect structures which cause flux-line pinning in the most perfect Y-, Nd-, and La-based single crystals are randomly distributed point defects and randomly spaced twin boundaries (TB's), which form a structure of planar defects parallel to the **c**-axis. The pinning properties of these two typical types of defects, however, are very different[1]. For TB's the pinning energy grows *linearly* with the length of the vortex, which is directed along the twin plane. This *strong* (due to *correlated* disorder) *anisotropic* pinning (because the pinning force is directed perpendicular to TB) has to be contrasted with *weak isotropic* pinning caused by point-like defects that compete with one another (due to *uncorrelated* disorder), leading to a mere *square-root* growth of the pinning energy with the length of the vortex line[1].

The most frequently used experimental way of studying the pinning-dependent vortex dynamics in layered HTSC's is the measurement of the temperature- and/or current-dependent in-plane longitudinal and transverse (Hall) magnetoresistivity in the geometry where the current density vector **j** is parallel to the layers of the HTSC sample, $\mathbf{H} \perp \mathbf{j}$ and directed under some angle ϕ with respect to the **c**-axis (**H** is the magnetic field vector). Note, however, that for $\mathbf{H}||\mathbf{c}$ the magnetoresistivity behavior is not influenced *by the internal anisotropy of the layered structure* and *the pinning anisotropy of TB's themselves* can be tested only for parallel twin planes because for randomly distributed TB's (as well as for purely point-like pins) the in-plane resistive response is isotropic.

The magnetoresistivity tensors for both types of disorder in $\mathbf{H}||\mathbf{c}$ in-plane geometry have been studied theoretically. For the point pins it was shown that both, nonlinear longitudinal ($\rho_{||}$) and the Hall ($\rho_{\perp}$) magnetoresistivities are isotropic, and for $|\rho_{\perp}| << \rho_{||}$ the simple scaling relation $\rho_{\perp} \sim \rho_{||}^2$ exists[7]. For unidirected TB's it was shown that the anisotropy of the magnetoresistivity response strongly depends on the angle α between **j** and the direction of the TB's[8,9]. The physical origin of this pinning anisotropy is associated with the fact that the pinning force of the TB's is directed perpendicular to the twin planes, so that the vortices tend to move along these planes if the driving force has a nonzero component in any in-plane direction. Such a *guided motion* of vortices in a pinning potential of TB's leads to the appearance of a new (anisotropic) $\rho_{\perp}^{+}$ contribution to the transverse (with respect to the **j**-direction) magnetoresistivity which is even with respect to magnetic field reversal. It may as well strongly modify the usual odd transverse Hall resistivity $\rho_{\perp}^{-}$, leading to a new anisotropic scaling relation between the components of the anisotropic magnetoresistivity tensor[9].

As far as the analysis of existing experimental data is concerned, none of the present theoretical studies in the limiting cases of purely anisotropic or isotropic pinning are sufficient. The more general approach is needed.

The objective of this paper is to present results of a theory for the calculation of the nonlinear magnetoresistivity tensor at arbitrary value of competition between point-like and anisotropic planar disorder for the case of in-plane geometry of experiment.

2. MAIN RELATIONS

2.1. Formulation of the problem

The Langevin equation for a vortex moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B} = \mathbf{n}B$ ($B \equiv |\mathbf{B}|$, $\mathbf{n} = n\mathbf{z}$, $\mathbf{z}$ is the unit vector in the z direction, and $n = \pm 1$) has the form

$$\eta \mathbf{v} + n\alpha_H \mathbf{v} \times \mathbf{z} = \mathbf{F}_L + \mathbf{F}_p^i + \mathbf{F}_p^a + \mathbf{F}_{th} \tag{1}$$

where $\mathbf{F}_L = n(\phi_0/c)\mathbf{j} \times \mathbf{z}$ is the Lorentz force (ϕ_0 is the magnetic flux quantum, c is the speed of light), $\mathbf{F}_p^a = -\nabla U_p$ is the anisotropic pinning force ($U_p(x)$ is the pinning potential), $\mathbf{F}_p^i$ is the isotropic pinning force induced by uncorrelated point-like disorder, $\mathbf{F}_{th}$ is the thermal fluctuation force, η is the vortex viscosity, and α_H is the Hall constant. The fluctuation force $\mathbf{F}_{th}(t)$ is represented by a Gaussian white noise whose stochastic properties are specified by the relations $\langle F_{th,i}(t)\rangle = 0$ and $\langle F_{th,i}(t)F_{th,j}(t')\rangle = 2T\eta\delta_{ij}\delta(t-t')$, where T is the temperature in energy units.

2.2. Anisotropic pinning

In the absence of point-like disorder (i.e. $\mathbf{F}_p^i = 0$), a Fokker-Planck equation consistent with Eq. (1) was solved in[9]. As a result, rather simple formulas were derived for the experimentally observable nonlinear even and odd longitudinal and transverse dimensionless magnetoresistivities $\rho^{\pm}_{\|,\perp}(j,\theta,\alpha,\varepsilon_a)$ as functions of the transport current density j, dimensionless temperature θ and relative volume fraction $0 \le \varepsilon_a \le 1$ occupied by the parallel TP's directed at an angle α with respect to the current direction:

$$\rho^+_{\|a} = \nu_a^+ \cos^2\alpha + \sin^2\alpha, \qquad \rho^+_{\perp a} = (\nu_a^+ - 1)\sin\alpha\cos\alpha, \tag{2}$$

$$\rho^-_{\|a} = \nu_a^- \cos^2\alpha, \qquad \rho^-_{\perp a} = n\epsilon\nu_a^+ + \nu_a^- \sin\alpha\cos\alpha. \tag{3}$$

Here $\epsilon \equiv \alpha_H/\eta << 1$, $\nu_a = \nu_a(F)$ where $F \equiv F_{Lx} - n\epsilon F_{Ly}$, and $\nu_a^{\pm}(n) \equiv [\nu_a(n) \pm \nu_a(-n)]/2$ are the even (+) and odd (−) parts of the function $\nu_a(j,\theta,\alpha,\varepsilon_a)$ which can be considered as the probability of overcoming the potential barrier of the twins in the x direction under the influence of the effective force F (see[9]). This ν_a-function describes an essentially nonlinear transition from the linear low-temperature thermoactivated flux flow (TAFF) regime of vortex motion to the ohmic flux flow (FF) regime. It is a step-like function of j or θ for small temperature or current density (see Figs. 4, 5 in[9]).

It follows from Eqs. (2)-(3) that for $\alpha \neq 0, \pi/2$ the observed resistive response contains not only the ordinary longitudinal $\rho^+_{||a}(\alpha)$ and transverse $\rho^-_{\perp a}(\alpha)$ magnetoresistivities, but also two new components induced by the pinning anisotropy: an *even transverse* $\rho^+_{\perp a}(\alpha)$ and an *odd longitudinal* component $\rho^-_{||a}(\alpha)$. The physical origin of the $\rho^+_{\perp a}(\alpha)$ (which is independent of ϵ) is related in an obvious way with the guided vortex motion along the "channels" of the washboard pinning potential in the TAFF regime. On the other hand, the component $\rho^-_{||a}(\alpha)$ is proportional to the odd component ν_a^-, which is zero at $\epsilon = 0$ and has a maximum in the region of the nonlinear transition from the TAFF to the FF regime at $\epsilon \neq 0$ (see Figs. 6, 7 in[9]). The (j, θ)-dependence of the odd transverse (Hall) resistivity $\rho^-_{\perp a}(j, \theta)$ has contributions both, from the even $\nu_a^+ \approx \nu_a$ and from the odd ν_a^- components of the $\nu_a(j, \theta)$-function. Their relative magnitudes are determined by the angle α and the effective Hall constant ϵ. Note, that as the odd longitudinal $\rho^-_{||a}$ and odd transverse $\rho^-_{\perp a}$ magnetoresistivities arise by virtue of the Hall effect, their characteristic scale is proportional to $\epsilon << 1$ (see Eqs. (3)).

2.3. Isotropic pinning

For purely isotropic pinning (i.e. $\mathbf{F}_p^a = 0$), Eq. (1) can be solved[7] using the fact that $\mathbf{F}_p^i = -\gamma(v)\mathbf{v}$ where $\gamma(v)$ is the velocity-dependent viscosity and $v \equiv |\mathbf{v}|$. Then in terms of the probability function of overcoming the effective potential barrier of isotropic pinning $\nu_i(j, \theta, \varepsilon_i)$ which is simply related to $\gamma(v)$ (see, for example[10]), experimentally observable nonlinear even longitudinal $\rho_{||i}$ and odd transverse $\rho_{\perp i}$ magnetoresistivities can be derived at $\epsilon << 1$ as functions of $j, \theta, \epsilon, \varepsilon_i$:

$$\rho_{||i} = \nu_i(F_i), \quad \rho_{\perp i} = n\epsilon\nu_i^2(F_i), \quad F_i \equiv |\mathbf{F}_L| \tag{4}$$

From Eq. (4) the well-known scaling relation $\rho_{||i} \sim (\rho_{\perp i})^2$ follows[7].

2.4. Competition between anisotropic and isotropic pinning

In this sub-section I present the results of a generalized approach for the calculation of the anisotropic nonlinear magnetoresistivities $\rho^{\pm}_{||,\perp}$ for any arbitrary value of competition between point-like and anisotropic planar disorder in the in-plane geometry. As it follows from the solution of Eq. (1)

$$\rho_{\|}^{+} = \rho_{\|i} \cdot \rho_{\|a}^{+}, \quad \rho_{\perp}^{+} = \rho_{\|i} \cdot \rho_{\perp a}^{+}, \tag{5}$$

$$\rho_{\|}^{-} = \nu_i^{-} \rho_{\|a}^{+} + \rho_{\|i} \cdot \rho_{\|a}^{-}, \tag{6}$$

$$\rho_{\perp}^{-} = \nu_a \rho_{\perp i} + \rho_{\|i} \cdot \rho_{\|a}^{-} \tan\alpha - \nu_i^{-}(1-\nu_a)\sin 2\alpha/2. \tag{7}$$

The magnetoresistivities $\rho_{\|,\perp a}^{\pm}$, $\rho_{\|,\perp i}$ and the $\nu_a \equiv \nu_a(F_{Lx})$-functions in Eqs. (5)-(7) are the same as those in subsections 2.2 and 2.3; $\nu_a^{-} = \nu_a^{-}(\tilde{F}_x)$, where $\tilde{F}_x \equiv F_{Lx} - n\epsilon\nu_i(F_a)F_{Ly}$, $F_a \equiv (F_{Ly}^2 + F_{Lx}^2\nu_a^2)^{1/2}$, $\nu_i^{-} \equiv [\nu_i(n) - \nu_i(-n)]/2 = \nu_i[F_a(n)]$ and $F_a(n) \equiv [F_{Ly}^2 + F_{Lx}^2\nu_a^2(\tilde{F}_{Lx}) + 2n\epsilon\nu_i(F_a)F_{Lx}F_{Ly}\nu_a(1-\nu_a)]^{1/2}$. It is easy to check that all previous results of sub-sections 2.2 and 2.3 follow from Eqs. (5-7) in the limits of purely anisotropic (i.e. $\nu_i = 1$, $\nu_i^{-} = 0$) and isotropic (i.e. $\nu_a = 1$, $\nu_a^{-} = 0$) pins.

3. MAIN RESULTS

3.1. General discussion

Eqs. (5)-(7) are accurate to first order in $\epsilon << 1$. They contain a lot of new physical information which will be elaborated on elsewhere[11]. Here, it must suffice to discuss in short the main physically important features of these equations. First of all, it follows that, the $\rho_{\|,\perp}^{\pm}$ magnetoresistivities can be found if the ν_a- and ν_i-functions are known. The converse statement is also valid: it is possible to reconstruct these functions from (j,θ,H)-dependent resistive measurements using only Eq. (5), where the Hall terms are ignored. Eqs. (6), (7), which arise due to the Hall effect, have a rather complicated structure which reflects a more pronounced competition between isotropic and anisotropic disorder in the Hall-mediated magnetoresistivities.

3.2. Point-like disorder and the guiding of vortices

For the discussion of the influence of point-like pins on the guiding of vortices in the anisotropic pinning potential it is sufficient to analyze Eqs. (5). Several main physical findings follow: a) the factor $\rho_{\|i}$ ensures that an anisotropic critical current density $j_c(\alpha,\theta)$ exists for arbitrary angles α(contrary to the results of sub-section 2.1), b) the angular dependence of the ratio $\rho_\perp/\rho_\|$, which determines the angle between $\mathbf{j}$ and $\mathbf{v}$ (see Eq. (36)

in[9]), is not influenced by the isotropic disorder, whereas c) the polar resistivity diagram $\rho(\alpha)$, which can be measured experimentally[2], is influenced by point-like pins because $\rho(\alpha) = \rho_{\|i}(\sin^2\alpha + \nu_a^2\cos^2\alpha)^{1/2}$.

3.3. New Hall voltages

As it follows from Eqs. (6) and (7), the odd longitudinal $\rho_{\|}^{-}$ and transverse $\rho_{\perp}^{-}$ magnetoresistivities contain terms with the ν_i^--function. They possess a highly anisotropic current- and temperature-dependent bump-like behavior. They tend to zero in the linear regime of vortex motion. For $\alpha = 0, \pi/2$ these new terms disappear, because $\nu_i^- = \nu_a^- = 0$ at these limits. The appearance of these new odd Hall contributions follows from the emergence of a certain equivalence of xy-directions due to a guiding of vortices along the channels of the washboard pinning potential for the case with $\alpha \neq 0, \pi/2$. Note also, that $\rho_{\|}^{-}$ includes two terms with similar signs, whereas in $\rho_{\perp}^{-}$ there are terms with opposite signs. The latter can give rise[11] to the well-known sign change in the (j, θ, H)-dependence of the Hall resistivity below T_c[7]. From Eqs. (5)-(7) new scaling relations (as for bianisotropic pinning[12]) for the Hall conductivity ϵ can be derived[11].

4. CONCLUSIONS

The analysis of formulas (5)-(7) in the form of graphs[11] gives a physically clear understanding of those (T, H, j, α) ranges and magnitudes of the internal parameters for which one of the competing types of disorder is dominant in in-plane magnetoresistive measurements in uniaxially anisotropic HTSC samples. New odd longitudinal and transverse Hall magnetoresistivities arise due to the presence of isotropic disorder. It should be noted that similar results may also be obtained within the frame of the phenomenological approach which takes into account the vortex-vortex interactions, ignored in the present model. In the stochastic approach, used here, one can calculate the nonlinear ν-functions for a given pinning potential[8–10] whereas in the phenomenological approach the $\nu(H, \theta, j)$ dependence cannot be calculated and must be determined (at $\epsilon << 1$) from experimentally obtained longitudinal and transverse resistivities by employing (5).

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