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Effect of self-heating on flux flow instability in a superconductor near T_c

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We have extended the Larkin–Ovchinnikov approach to the problem of viscous flux flow instability in "dirty" superconductors near T_c by taking into account quasiparticle heating, which is, in principle, unavoidable in finite magnetic fields in the case of real heat removal. According to the relationship between the magnetic field B value and the parameter B_T, introduced by us, there may be two qualitatively different mechanisms of instability: one due to heating of quasiparticles at $B \gg B_T$, and the other (considered by Larkin and Ovchinnikov) of nonthermal nature at $B \ll B_T$. The results obtained have allowed us to interpret some peculiarities in the resistive transition, observed experimentally for wide superconducting films. Based on resistive measurements, a new way of finding the quasiparticle energy relaxation time is proposed. In the case of perfect acoustic matching of the film and the substrate, the inelastic scattering times of quasiparticles on both the quasiparticles and the phonons can be obtained separately.

1. Introduction

The nonlinear behaviour of current–voltage characteristics (CVC) of superconducting films in the mixed state is generally analysed proceeding from the concepts of specific quasiparticle dynamics in the system of moving vortices as formulated by Larkin and Ovchinnikov (LO) [1]. According to these concepts, the electric field arising during the vortex motion accelerates the quasiparticles localized in a vortex core (as in a potential well) until their energy grows sufficiently for their escaping from the potential well. If in this case the time of quasiparticle energy relaxation τ_ϵ and the diffusion coefficient D are large enough, so that the diffusion length $l_\epsilon = (D\tau_\epsilon)^{1/2}$ substantially exceeds the core size, then the excitations can leave the core. The deficit of quasiparticles in the cores will be the greater, the higher the vortex velocity is. As a result, as the vortex velocity v increases, the viscous damping coefficient η decreases or, in other words, the film resistivity grows with growing electric field value [#1].

[#1] Further on, we shall assume the superconductor temperature T to be close to the critical temperature T_c, since at $T_c - T \ll T_c$ the nonlinear effects considered in the paper are most pronounced.

Experimentally, for current-biased operation at not too high magnetic fields ($B \lesssim 0.4 H_{c2}$) the nonlinear resistive part of the CVC usually shows a jumplike voltage rise (e.g., see refs. [2] and [3]). According to LO, this jump results from the instability homogeneous flux flow at a critical vortex velocity

$$v^* = 1.02 (D/\tau_\epsilon)^{1/2} (1 - T/T_c)^{1/4} \quad (1)$$

when the Lorentz force equals the greatest damping force value. (The velocity dependence of the damping force $F_v(v) = \eta(v) v$ has the shape of a curve with a maximum due to a drastic decrease in the viscous damping coefficient $\eta(v)$ at $v \gg v^*$.)

The values of the inelastic scattering time of quasiparticles found experimentally [2,3] on the basis of expression (1) for In, Sn, Al are equal in order of magnitude to the values obtained in the independent experiments and to theoretical estimates. The authors of refs. [2] and [3] have established, however, some peculiarities in their experiments that are inconsistent with the conclusions following from the LO theory.

Thus, Huebener et al. [3] have indicated an anomalous dependence of the inelastic scattering time on the applied magnetic field value. Another essential discrepancy between theory and experiment is an appreciable experimental decrease in the instability current value I^*, whereas, as follows from ref. [1], the instability current value, at least, in low fields ($B \ll H_{c2}$) should not depend on B.

Here we demonstrate that the above-mentioned discrepancies between theory and experiment for the finite vortex density may have a common cause, namely, heating of quasiparticles, which is, in principle, inherent in experiment because of the finite rate of removing the power dissipated in the sample.

It is physically evident that the heat removal process includes two stages:

(1) the energy transfer from quasiparticles to phonons through radiation of nonequilibrium phonons and

(2) the flow of heat from the film to the substrate as a result of their mutual phonon exchange. As follows from the work of one of the co-authors [4], one can distinguish two different cases of heat removal in accordance with the relationship between the free path length $l_{pe}(T)$ of thermal phonons with respect to scattering by electrons ($l_{pe}(T) \sim \hbar v_F / k_B T$, from which $l_{pe} \sim 10^{-3}$ to 10^{-4} cm, if the Fermi velocity $v_F \sim 10^8$ cm/s and $T \sim 1$ to 10 K) and the so-called effective film thickness $d_{ef} = d/\alpha$ (where d is the film thickness, and α is the mean probability of phonon transmission from the film to the substrate, $0 < \alpha < 1$). If $l_{pe} \ll d_{ef}$, then the nonequilibrium phonons are reabsorbed in the film by quasiparticles, so that for both systems we have the same temperature, which is higher than the substrate (bath) temperature T_0 (Joule heating regime). Typically, the heat removal rate is determined here by the film–substrate interface transparency to phonons [5]. If the film thickness $d \ll l_{pe}$, then by improving the acoustic transparency fo the interface we can come to the opposite limiting case $d_{ef} \ll l_{pe}$ (which was called in ref. [4] the electron overheating regime) when nonequilibrium phonons leave the film without being reabsorbed and the heat removal rate is determined by the value of the electron–phonon coupling constant. Yet, in this case, too, the heat removal rate remains finite.

Section 2 of this paper presents a detailed calculation of heat removal from the film to the single-crystal substrate (where ballistic propagation of phonons is realized) in the general case, i.e., at an arbitrary acoustic transparency of the film–substrate interface. The heat removal model used in our calculations corresponds to the so-called acoustic mismatch theory of adjoining materials and makes it possible to derive, in principle, the heat transfer coefficient (see formula (19)) for an arbitrary relationship between d_{ef} and $l_{pe}(T)$ [#2].

The main relations describing the vortex viscous flow instability in a "dirty" film near the critical temperature are derived in section 3. The same section describes the correlation between the flux flow instability threshold values and the inelastic quasiparticle–quasiparticle and quasiparticle–phonon scattering rates.

Section 4 comprises the discussion of the results and the analysis of experiments. In section 5 we summarize the work and formulate the conclusions.

2. Microscopic analysis of heat removal from a thin film in the resistive state

As the direct transport current flows through the film being in the mixed state, it causes power dissipation $P = \rho_f J^2$ in the unit volume (J is the current density, ρ_f the flux flow resistivity). To calculate the heating effect, we use the electron temperature approximation. In the case when the requirement of quasiparticle gas thermalization is fulfilled (e.g., due to frequent electron–electron collisions), this approximation is justified. But it can also be admissible even if electron–phonon collisions dominate, since (as demonstrated for a normal metal [7]) the basic difference between the exact nonequilibrium quasiparticle distribution function and the Fermi distribution with an effective temperature lies in the energy range $\epsilon \gg T$, while the conductivity of the mixed-state film near T_c is mainly contributed by the energy range of the order of the superconducting gap value $\Delta \ll T_c$ [1]. The electron temperature, being uniform throughout the film thickness due to a high electron thermal conductivity [4], can be found from the heat balance equation. The electron temperature uniformity in the film plane is due to an approximately uniform distribution of the above-gap quas-

[#2] Note that the acoustic mismatch theory has been reliably confirmed in a series of experimental works by Weiss and Coworkers (e.g., see ref. [6] and the references cited there).

iparticles in this plane (because of an essentially greater length of quasiparticle energy relaxation in comparison with the intervortex distance).

The process of heat removal can be quantitatively analysed on the basis of the kinetic equation for the phonon distribution function. If the z-axis is normal to the film, with $z=0$ coinciding with the film–substrate interface, and $z=d$ with the free boundary of the film, then the kinetic equation takes the form

$$s_z \frac{\partial}{\partial z} n(\boldsymbol{q}, z) = I_{\text{pe}} , \quad (2)$$

where $\boldsymbol{q}$ is the phonon wavevector, and s_z is the phonon velocity projection onto the z-axis. The collision integral,

$$I_{\text{pe}} = -\frac{m^2\mu^2}{8\pi\hbar^4\rho s} \int \int_{-\infty}^{\infty} \mathrm{d}\xi\,\mathrm{d}\xi'$$
$$\times\Big\{2\Big(1-\frac{\Delta^2}{EE'}[(f_E-f_{E'})n-f_E+f_Ef_{E'}]$$
$$\delta(E-E'+\hbar\omega_q)+(1+\Delta^2/EE')$$
$$\times[(1-f_E-f_{E'})n-f_Ef_{E'}]\delta(E+E'-\hbar\omega_q)\Big\} \quad (3)$$

is written in the Debye model for phonons and in the approximation of the isotropic quadratic $\epsilon(\boldsymbol{p})$-dependence for electrons (e.g., see ref. [8]). The electron–phonon interaction (EPI) is described in the deformation-potential approximation. In expression (3) μ is the EPI constant of the order of the Fermi energy, m is the effective electron mass, ρ the film density, s the longitudinal sound velocity in the film, $E=[\xi^2+\Delta^2(T)]^{1/2}$ the quasiparticle energy, $\omega_q=qs$ is the dispersion relation for phonons [#3]. The form of the collision integral I_{pe} is substantially simplified if the quasiparticle distribution function is taken in the effective-temperature approximation. Really, for $f_E=[1+\exp(E/k_BT)]^{-1}$ we have

$$I_{\text{pe}} = -\frac{1}{\tau_{\text{pe}}}[n(\boldsymbol{q}, z)-N_q(T)] , \quad (4)$$

where $N_q(T)$ is the Bose distribution function at temperature T; and for τ_{pe} we obtain the known (e.g., see ref. [9]) expression $\tau_{\text{pe}}^{-1}=\tau_{\text{ps}}^{-1}+\tau_{\text{pb}}^{-1}$, where τ_{ps} is the phonon lifetime relative to the scattering by quasiparticles

$$\tau_{\text{ps}}^{-1} = \frac{m^2\mu^2}{\pi\hbar^4\rho s}$$
$$\times\int_{\Delta}^{\infty} \mathrm{d}E \frac{E(E+\hbar\omega)-\Delta^2}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}$$
$$\times(f_E-f_{E+\hbar\omega}) , \quad (5)$$

and τ_{pb} is the lifetime of the phonon of energy $\hbar\omega>2\Delta$ referring to the breaking and creation of Cooper pairs

$$\tau_{\text{pb}}^{-1} = \frac{m^2\mu^2}{2\pi\hbar^4\rho s}\int_{\Delta}^{\hbar\omega-\Delta} \mathrm{d}E$$
$$\times\frac{E(\hbar\omega-E)+\Delta^2}{\sqrt{E^2-\Delta^2}\sqrt{(\hbar\omega-E)^2-\Delta^2}}$$
$$\times(1-f_E-f_{\hbar\omega-E}) . \quad (6)$$

The solution of eq. (2) with the collision integral (4) is given by

$$n^{\gtrless}(\boldsymbol{q}, z)=C^{\gtrless}\exp(\mp z/l)+N_q(T) . \quad (7)$$

Here $n^{\gtrless}(\boldsymbol{q}, z)$ are the distribution functions of phonons with positive (negative) wave vector projections onto the z-axis, $l=|s_z|\tau_{\text{pe}}$ and the integration constants C should be found from boundary conditions. If $\alpha(\theta)$ is the transmission probability for the phonon incident at an angle θ to the film–substrate interface [#4], while at the other film boundary the phonons are specularly reflected, then the following conditions should be fulfilled at these boundaries:

$$n^{>}(\boldsymbol{q}, 0)=\alpha N_q(T_0)+\beta n^{<}(\tilde{\boldsymbol{q}}, 0) , \quad (8)$$

$$n^{<}(\tilde{\boldsymbol{q}}, d)=n^{>}(\boldsymbol{q}, d) . \quad (9)$$

Her $\beta=1-\alpha$, $\tilde{\boldsymbol{q}}=(\boldsymbol{q}_{\parallel}, -q_z)$, whereas $\boldsymbol{q}=(\boldsymbol{q}_{\parallel}, q_z)$. The substitution of eq. (7) into the boundary conditions gives

[#3] In expression (3) we have neglected the variations of the order parameter and the phonon scattering by quasiparticles entrapped in vortex cores, since near T_c the phonons are predominantly absorbed by uniformly distributed above-gap quasiparticles.

[#4] For the coefficient of phonon transmission from side 1 to side 2, the acoustic mismatch theory [5] uses the expression $\alpha(\theta_1)=4(\rho_2s_2/\rho_1s_1)(\cos\theta_2/\cos\theta_1)$ $(\rho_2s_2/\rho_1s_1+\cos\theta_2/\cos\theta_1)^{-1}$ where the angles θ_1 and θ_2 are related by Snell's law $s_2\sin\theta_1=s_1\sin\theta_2$.

$$C^{>} = x^{-2} C^{<} = \alpha(1-\beta x^2)^{-1} [N_q(T_0) - N_q(T)] . \quad (10)$$

The quantity $x = \exp(-d/l)$ is the probability for the phonon to propagate (with a given velocity projection s_z) from one film boundary to the other without being absorbed. Knowing the phonon distribution function at $z=0$, it is not difficult to write the expression for the heat flow from the film to the substrate

$$Q = \sum_{q_1, q_z<0} \hbar\omega_q |s_z| \tilde{\alpha} [N_q(T) - N_q(T_0)] , \quad (11)$$

where $\tilde{\alpha} = \alpha(1-x^2)/(1-\beta x^2)$ is the effective transparency of the film–substrate interface, and V the film volume. In formula (11) it is convenient to change summation by integration

$$Q = \hbar s^2 (2\pi)^{-2} \int_0^{q_D} \mathrm{d}q\, q^3 \tilde{\alpha}(q) [N_q(T) - N_q(T_0)] . \quad (12)$$

In this case, the angle-averaged transparency is

$$\bar{\alpha}(q) = \int_0^{\pi/2} \tilde{\alpha}(\theta, q) \cos\theta \sin\theta \,\mathrm{d}\theta . \quad (13)$$

Let us now analyse the limiting heat removal regimes. At temperatures of a superconductor close to critical temperature, the characteristic phonon energy si significantly higher than the superconducting energy gap Δ; therefore, the lifetime of thermal phonons in the superconductor can be taken as the lifetime of phonons in the normal state at $\hbar\omega_q \sim k_B T$ [4]:

$$\tau_{pe} = \frac{2\pi\hbar^3 \rho s}{\mu^2 m^2 \omega_q} \sim \frac{v_F}{s\omega_q} . \quad (14)$$

If the film thickness $d \gg l_{pe}$ ($l_{pe} \sim \hbar v_F / k_B T$), then for the phonons giving the main contribution to the heat flow Q, the x value is exponentially small and $\tilde{\alpha} = \alpha$. For the films with $d \ll l_{pe}$ the effective transparency is

$$\tilde{\alpha} = (2d/l)(1 + 2\beta d_{ef}/l)^{-1} , \quad (15)$$

from which it follows that in a thin film two limiting cases can be realized, depending on the value of the parameter

$$\delta = 2\beta d_{ef}/l . \quad (16)$$

If in consequence of a low phonon transparency of the interface we have $\delta \gg 1$, then the effective transparency $\tilde{\alpha} = \alpha$ (as in the case of $d \gg l_{pe}$). Calculation of the heat flow at $\tilde{\alpha} = \alpha$ gives the known result of the Little theory [5]

$$Q = \frac{\pi^2}{120} \bar{\alpha} \frac{k_B^4}{\hbar^3 s^2} (T^4 - T_0^4) . \quad (17)$$

In expression (17) the temperature refers both to quasiparticles and phonons, since the reabsorption of nonequilibrium phonons in the film leads to their thermalization.

In the opposite limit, $\delta \ll 1$, the effective transparency $\tilde{\alpha}$ is equal to $2d/l$ and is independent of the interface properties. This limit corresponds to the situation when nonequilibrium phonons escape from the film without reabsorption. In this case the heat flow to the substrate (with an accuracy to the corrections of the order of $(\Delta/T)^2 \ll 1$) is given by the expression derived by Kaganov et al. [10]:

$$Q = \frac{2D_5}{(2\pi\hbar)^3} \frac{m^2 \mu^2 d}{\hbar^4 \rho s^4} k_B^5 (T^5 - T_0^5) , \quad (18)$$

where $D_5 = \int_0^\infty x^4 (e^x - 1)^{-1} dx$ .The heating regime of the film corresponding to this limit can be called the electron overheating, because the escape of nonequilibrium phonons from the film permits us to characterize the quasi-particles and the lattice by different temperatures, T and T_0, respectively.

Since the film heating in the resistive state near T_c is not usually too large, i.e., $T - T_0 \ll T_0$, the heat flow from the film to the substrate can be written in the linearized form as

$$Q = h(T_0)(T - T_0) \quad (19)$$

and, as follows from formulae (17) and (18), the expressions

$$h_J(T_0) = \frac{\pi^2}{30} \bar{\alpha} \frac{k_B^4 T_0^3}{\hbar^3 s^2} \quad (20)$$

and

$$h_e(T_0) = \frac{10 D_5}{(2\pi\hbar)^3} \frac{m^2 \mu^2 d}{\hbar^4 \rho s^4} k_B^5 T_0^4 \quad (21)$$

are valid for the heat transfer coefficient $h(T_0)$ in the limiting cases of the Joule heating and electron overheating, respectively. Note, that the coefficient h_e given by eq. (21) can also be represented as

$$h_e(T_0)=\frac{c_e(T_0)d}{0.22\tau_{ep}(T_0)}, \tag{22}$$

where the electron specific heat of the unit volume $c_e=(\pi^2/3)k_B^2N(0)T_0$ ($N(0)=m^2v_F/\pi^2\hbar^3$ is the density of states on the Fermi surface), and the time of inelastic scattering of the electrons with energy $\epsilon \ll T_0$ due to their scattering by the thermalized phonons having temperature T_0 is given by

$$\tau_{ep}(T_0)=\frac{(2\pi)^3N(0)\hbar^7\rho s^4}{14\zeta(3)m^2\mu^2(k_BT_0)^3}. \tag{23}$$

Expression (23) is obtained from the standard electron–phonon collision integral, if one assumes, as in refs. [1], [10] that the EPI is described in terms of the deformation potential.

3. Viscous flux flow instability with consideration of the finite heat removal rate

As mentioned in the introduction, the instability of a uniform vortex flow in a superconducting film is due to a nonlinear dependence of the flux flow conductivity on the electric field. The expression for the conductivity in the "dirty" limit near T_c has been derived by LO [1]:

$$\sigma(E)=\sigma_nH_{c2}(T)B^{-1}(1-T/T_c)^{-1/2}\times[1+(E/E^*_{LO})^2]^{-1}f(B/H_{c2}). \tag{24}$$

Here σ_n is the normal-state film conductivity, E is the electric field, $E^*=v^*B/c$ (c is the speed of light). The function $f(B/H_{c2})$ which takes into account the vortex core overlap, has been tabulated in ref. [11]. For the magnetic field values of interest ($B\lesssim 0.4H_{c2}$) the function $f(B/H_{c2})$ is about 4.04 [11,12].

One should bear in mind that the quasiparticle temperature involved in eq. (24) is dependent on the electric field value and should be found from the heat balance relation $Q=Pd$. Denoting the temperature and the electric field that correspond to the threshold of flux flow instability by T^* and E^*, respectively, we can write for them the set of equations including the heat balance equation

$$h(T_0)(T^*-T_0)=d\sigma(E^*)(E^*)^2 \tag{25}$$

and the CVC extremum condition

$$\frac{d}{dE}[\sigma(E)E]_{E=E^*}=0. \tag{26}$$

If then we introduce the dimensionless variables $e=E^*/E^*_{LO}(T_0)$, and $t=(T_c-T^*)/(T_c-T_0)$, and for the upper critical field entering into $\sigma(E)$ we use the known expression

$$H_{c2}(T)=\frac{4\phi_0}{\pi^2\hbar D}k_B(T_c-T), \tag{27}$$

which is valid for superconductors with a short free path (ϕ_0 is the magnetic flux quantum), the set of eqs. (25) and (26) can be written as

$$1-t=2bte^2/(e^2+\sqrt{t}), \tag{28}$$

$$1+\frac{e}{2t}\frac{dt}{de}-\frac{e}{\sqrt{t}}\left(1-\frac{e}{t}\frac{dt}{de}\right)=0, \tag{29}$$

where $b=B/B_T$ is the dimensionless magnetic field. The physical meaning of the parameter

$$B_T=0.374\,k_B^{-1}ce_0R_\square h\tau_\epsilon, \tag{30}$$

which plays an important role in the theory, will be clarified below on analysing the limiting cases of high and low magnetic fields. The parameter $R_\square=(\sigma_nd)^{-1}$ from eq. (30) is the sheet resistance, and e_0 is the electron charge.

The set of eqs. (28, 29) has the exact solution

$$t=[1+b+(b^2+8b+4)^{1/2}]/3(1+2b), \tag{31}$$

$$e^2=(1/2)\sqrt{t}(3t-1). \tag{32}$$

From eqs. (31) and (32) we obtain the following expression for the threshold electric field E^*:

$$\frac{E^*}{E_0}=\frac{(1-t)(3t+1)}{2\sqrt{2}t^{3/4}(3t-1)^{1/2}}, \tag{33}$$

with the normalizing parameter

$$E_0=1.02(B_T/c)(D/\tau_\epsilon)^{1/2}(1-T_0/T_c)^{1/4} \tag{34}$$

which is independent of the magnetic field.

The current density $J^*=\sigma(E^*)E^*$, corresponding to the instability threshold, can also be expressed in terms of t as

$$\frac{J^*}{J_0}=\frac{2\sqrt{2}t^{3/4}(3t-1)^{1/2}}{3t+1}. \tag{35}$$

Here we have introduced the normalizing current density

$$J_0 = 2.62(\sigma_n/e_0)(D\tau_\epsilon)^{-1/2} \times k_B T_c(1-T_0/T_c)^{3/4} \quad (36)$$

which, as one can verify, is coincident with the current J^* value at $E^*=0$.

Relationships (33) and (35) are the parametric form of the curve $E^*=E^*(J^*)$ passing through the instability points for the series of the CVC taken at different applied magnetic field values and fixed bath temperature. This curve and the instability points obtained in the experiment with an In film [3] are shown in fig. 1.

The dependence fo the threshold E^* and J^* values on the magnetic filed is given by formulae (33) and (35), where the parameter t must be expressed in terms of the magnetic field by the use of eq. (31). The corresponding curves are shown in figs. 2 and 3.

Turning to the analysis of the expressions derived, we note that according to formula (31) the film temperature at the instability point is the increasing function of the magnetic field with two different regions of $T^*(B)$ behaviour. In the magnetic fields $B \ll B_T$, where the vortex density is not large, and the film heating is small, the temperature T^* increases proportionally to B. For $B \gg B_T$ (but $B \lesssim 0.4H_{c2}$), the temperature T^* approaches the constant value $T_c-(1/3)(T_c-T_0)$, to which the value

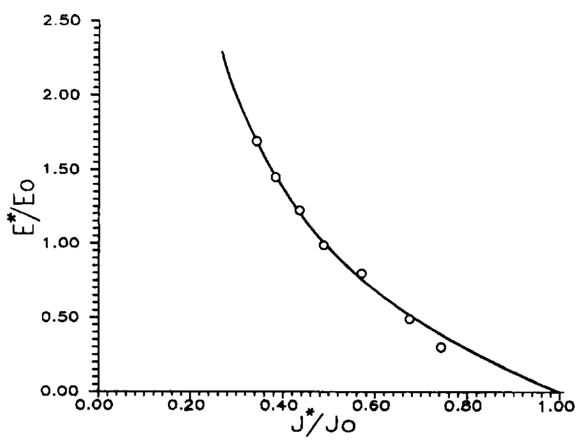

Fig. 1. Comparison of the theoretical curve $E^*(J^*)$ (solid line) and the experimental data. The open circles correspond to the E^* and J^* values for the indium film In-10-b [3].

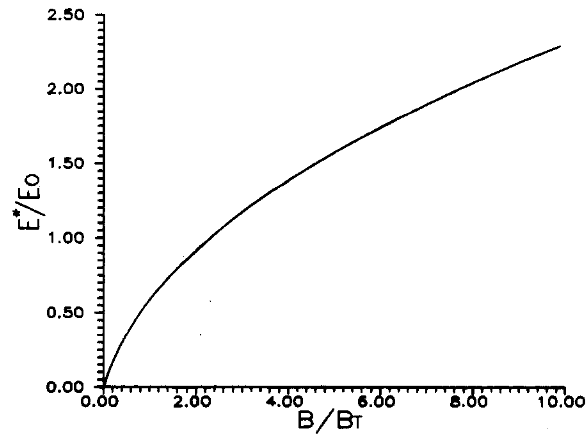

Fig. 2. The nonlinear magnetic field dependence of the electric field E^* associated with instability point of the CVC. The parameters B_T and E_0 are given by eqs. (30) and (34), respectively.

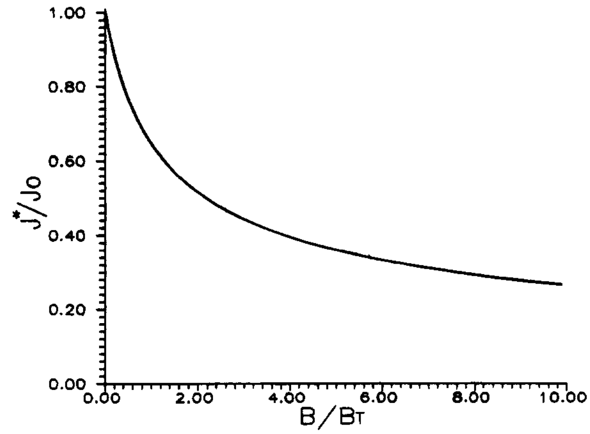

Fig. 3. The current density J^* corresponding to the onset of the flux flow instability as a function of the magnetic field. The decrease of J^* is due to self-heating under viscous flux flow. The parameter J_0 is determined by eq. (36).

of dissipated power equal to $(2/3)J_0E_0$ corresponds.

A more detailed analysis for the fields $B \ll B_T$ can be carried out by expanding the function $t(b)$ (formula (31)) in a power series with $b \ll 1$. The substitution of the zero power approximation, $t=1$, into eq. (32) shows that in this approximation the threshold electric field value coincides with the result obtained by LO [1]: $E^*(T^*)=E^*_{LO}(T_0) \backsim B$. At the same time, the film heating associated with the following expansion terms leads to the deviation of

$E^*(B)$ from a linear dependence (see fig. 2), and to the decrease in conductivity σ. As a result, the threshold current density J^* value decreases with a growing magnetic field (see fig. 3), in contrast to the result $J^*=\text{const}$ following from ref. [1].

To elucidate the physical meaning of the parameter B_T, we turn to the case of a heat removal, where B_T is so small that $B \gg B_T$, provided that the condition $B \lesssim 0.4H_{c2}$ is satisfied. In this limit $t(b)$ can be expanded in powers of $b^{-1} \ll 1$ as

$$t=(1/3)[1+(2/b)]\,. \tag{37}$$

Hence, in accordance with formula (33), we have

$$E^*=\left(\frac{hBT_c}{2.02\,\sigma_n dH_{c2}(0)}\right)^{1/2}\left(\frac{T_c-T_0}{3T_c}\right)^{1/4}. \tag{38}$$

Note that now E^* is independent of the time of quasiparticle energy relaxation, which plays an important role in the mechanism of viscous flux flow instability described in the introduction. Moreover, as can be easily seen, due to the smallness of $b^{-1/2} \ll 1$, the threshold electric field E^*, involved in eq. (38), is small in comparison with $E^*_{LO}(T^*)$. Taken together these facts are indicating a different nature of instability at $B \gg B_T$. Actually, it can be demonstrated that in the case of weak heat removal ($B \gg B_T$) the flux flow instability is mainly due to the decrease in conductivity because of a rising temperature in low electric fields $E \ll E^*_{LO}$, when the nonlinearity of $\sigma(E)$, considered by LO [1], can be neglected. Thus, the field B_T introduced above separates the regions, where the nonthermal or pure heating mechanisms of the instability dominate.

To conclude this section, we shall discuss in greater detail the time τ_ϵ, which is the energy relaxation time of the electron of energy $\epsilon \ll T$. In ref. [1], LO have suggested that the energy relaxation of quasiparticles should be due to their scattering by phonons. In this case, τ_ϵ is coincident with the electron–phonon collision time τ_{ep}. A similar calculation, which we made for a more general case, including both electron–phonon and electron–electron collisions, has revealed that the two mechanisms of inelastic quasiparticle scattering contribute additively to the energy relaxation rate, and so, the time τ_ϵ comprised in the formulae of this section, is given by

$$\tau_\epsilon(\tau_{ep}^{-1}+\tau_{ee}^{-1})^{-1}\,, \tag{39}$$

where τ_{ee}^{-1} is the inelastic electron–electron scattering rate.

4. Discussion and comparison with experiment

Here, primarily emphasis will be given to the problem of what parameters characterizing the film or the film–substrate thermal coupling can be extracted from the CVC measured in the experiment. These parameters are first of all the time of quasiparticle energy relaxation τ_ϵ and the heat transfer coefficient h. Their values can be estimated from the series of CVC measured at different applied magnetic field values ($B \lesssim 0.4H_{c2}$) and a fixed bath temperature. In fact, if by an appropriate choice of E_0 and J_0 the calculated $E^*(J^*)$ curve is made to coincide with the instability points for given CVC series (see fig. 1), then by using the J_0 value it is possible to calculate the inelastic relaxation time τ_ϵ, provided that the remaining parameters entering into eq. (36) are known from independent experiments. Then, based on formulae (34) and (30) and the known E_0 value we can calculate the heat transfer coefficient h.

The substitution of h and τ_ϵ values into formula (30) allows us to obtain the parameter B_T, which is of importance for interpreting the experiment, because for magnetic fields $B \ll B_T$ the instability of uniform flow is essentially due to the mechanism considered by LO [1], while for fields $B \gg B_T$, the dominant mechanism of the instability is the decrease in the flux flow conductivity because of the heating of quasiparticles. Note that the B_T value can be found in another independent way if the current J^* measured as a function of the magnetic field is made to fit the theoretical curve $J^*(B)$ (fig. 3). In this case, naturally, the calculated $E^*(B)$ curve (fig. 2) must also coincide with the correspondent dependence derived from the experiment.

In the electron overheating regime there is, in principle, an interesting possibility of determining separately the times of inelastic electron collisions with electrons and phonons, provided that the τ_ϵ and h_e values are preliminarily found. Since under this regime the heat transfer coefficient h_e involves the time τ_{ep}, relations (22) and (39) can be considered as a set of equations with respect to τ_{ee} and τ_{ep}.

Note also that in the Joule heating regime the heat transfer coefficient h_J is independent of τ_{ep} (see formula (20)); therefore, according to eq. (39), τ_{ep} or τ_{ee} can be derived through the τ_ϵ value only if electron–phonon or electron–electron collisions are dominant, respectively.

On realizing electron overheating experimentally, one should bear in mind that many commonly used substrates, such as, e.g., sapphire or quartz, in most cases do not provide a good acoustic match with superconducting films, with the result that the phonons emitted by quasiparticles are multiply reflected from the film–substrate interface and are reabsorbed in the film. At the same time, there exist such combinations of a film and a substrate, that the probability of phonon transmission across the interfaces is close to unity. To give examples, we mention the known pair of Al on the BaF_2 substrate [13] or Sn on the CdTe substrate used by Skocpol [14]. In these cases, if the condition $d < l_{pe}$ is satisfied, practically all phonons, emitted by "hot" quasiparticles penetrate into the substrate without being reflected from the interface, and hence, we have here the electron overheating regime, where the heat removal rate is determined by the strength of EPI rather than by interface properties.

Now, in view of the concepts given above, we shall analyse the experimental data of ref. [3].

Rather good agreement between the theoretical curve $E^*(J^*)$ and the set of instability points for the CVC series fo the In-10-b film [3] (see fig. 1) leads us to the following estimates: $J_0 = 2.5 \times 10^5$ A/cm^2 and $E_0 = 14$ mV/cm. Given the J_0 value, it is easy to calculate the inelastic scattering time. For the above film, we have $\tau_\epsilon = 0.13$ ns at $T = 3.28$ K, this value being closer to 0.11 and 0.14 ns obtained in independent experiments [#5], than $\tau_\epsilon = 0.05$ ns derived from extrapolation of the function $\tau_\epsilon(B)$ to $B = 0$ [#6]. However, this difference can possibly be eliminated if one takes into account the magnetic field of the transport current, which may be of the same order of magnitude or even exceed the applied magnetic field value in the range of low fields.

[#5] See for the corresponding reference the list in ref. [3].

[#6] In our opinion, the magnetic filed dependence on the inelastic scattering time $\tau_\epsilon(B)$ in ref. [3] is probably due to the analysis of the experimental data on the basis of formulae from ref. [1], where the film heating has been neglected.

As mentioned above, the heat transfer coefficient h can be derived from the B_T value, providing that the E_0, τ_ϵ and the film parameters are known. The h value for the In-10-b film on a sapphire substrate [3] is estimated to be 0.1 W/cm^{2}K, which corresponds to $B_T = 1.8$ Oe and is apparently underestimated (see below).

In general, when interpreting the experiment one should bear in mind the existence of some factors which impede quantitative comparison between theory and experiment. For example, due to current fluctuations the CVC jump may occur before the instability point (where $dJ/dE = 0$) is reached. As a result, the parameter J_0 determined experimentally will be smaller than its calculated value. Besides, film inhomogeneity may also contribute to the stability threshold shift. Thus, the presence of a large number of pinning centres may enhance the stability of the flux flow regime, and this (opposite to the factor considered above) leads to increase in J_0. It is quite possible that in the case of strong inhomogeneities the stability of the viscous flux flow might first break locally along some channels lying across the film. It can be demonstrated that in this case the E_0 value would be underestimated and as a consequence, the heat transfer coefficient would be also undervalued.

5. Summary and conclusions

We have considered here the instability of a uniform viscous flux flow in isotropic "dirty" superconductors near T_c in finite electric and magnetic fields with due account of the finite rate of heat removal from the sample. In the framework of the acoustic mismatch theory we have analysed the problem of heat removal from a resistive state thin-film specimen on a massive dielectric substrate and have calculated the heat transfer coefficient for the limiting cases of Joule heating (bad acoustic match, formula (20)) and electron overheating (good acoustic match, formula (21)). It is demonstrated that two qualitatively different mechanisms of instability are possible, depending on the relationship between the magnetic field value B and the parameter B_T, the latter being proportional to the heat transfer coefficient h and the time of quasiparticle energy relaxation τ_ϵ (see formula (30)). If $B \ll B_T$

then the main cause of the flux flow instability is (in accordance with the LO concept) the nonthermal change of the distribution function of quasiparticles trapped in the vortex cores. In this case, an additional heating of the sample due to the finite heat removal rate is of minor importance. If, however, $B_T \ll B < H_{c2}$, then essentially a pure thermal instability of flux flow can occur in the electric field range $E \ll E^*_{LO}$, where the nonthermal change of the quasiparticle distribution function is of secondary importance.

It is also shown that the analysis of instability points on the CVC curves may give two parameters J_0 and E_0 (see formulae (36) and (34)), in terms of which we can express the electron energy relaxation rate τ_ϵ^{-1} (formula (39)) and the heat transfer coefficient h (formula (19)), and hence, the field B_T. The B_T value specifies the mechanism of instability.

Finally, we summarize the conclusions of this work, that may be most essential for experiment.

(1) For the finite vortex density, the finite heat removal rate leads to an unavoidable film overheating with respect to the bath (substrate) temperature. It is essential that the rate of heat removal from the film will be finite even for the ideal acoustic match with a massive single crystal substrate. The overheating value is determined in this case by the strength of electron–phonon coupling in the metal (formula (21)).

(2) Therefore, the LO formulae are valid only at a very low concentration of vortices. For the finite vortex density (finite magnetic field B values) it is necessary to take into account the power dissipation during the vortex motion and the film temperature rise [#7].

(3) The electron overheating regime offers the possibility of determining separately τ_{ep} and τ_{ee}. So, the study of flux flow instability near T_c may be still another way (in addition to the known ones [15]) of determining inelastic relaxation times of quasiparticles in superconducting metals.

[#7] Recent experimental results can be found in ref. [16].

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