Guiding of vortices and the Hall conductivity scaling in a bianisotropic planar pinning potential

Valerij A. Shklovskij
Institute for Theoretical Physics, National Science Center—Kharkov Institute of Physics and Technology, 61108, Kharkov, Ukraine and Kharkov National University, Physical Department, 61077, Kharkov, Ukraine

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The exactly solvable stochastic planar model of anisotropic pinning of vortices by two different orthogonally oriented washboard potentials at the temperature $T \geqslant 0$ and at arbitrary values of the vortex viscosity η and the Hall constant α_H is considered. The model describes nonlinear anisotropy effects caused, e.g., by the coexistence of intrinsic and twin-plane or grain-boundary pinning in layered high-T_c superconductors. Nonlinear guiding effects are discussed and the critical current anisotropy is analyzed. New anisotropic scaling relations for the Hall conductivity are predicted and the interrelation between the guiding of vortices and the Hall effect in nonlinear regimes is considered.

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Recently, the vortex dynamics in a planar pinning potential (PPP) has been extensively studied both theoretically[1–9] and experimentally.[10–16] Two main reasons stimulated these studies. First, in some high-T_c superconductors (HTSC's) twins can easily be formed during the crystal growth.[11–17] Second, in layered HTSC's the system of interlayers between parallel ab planes can be considered as a set of unidirectional planar defects which provoke the intrinsic pinning of vortices.[1]

As the pinning force in a PPP is directed perpendicular to the pinning planes,[1] the vortices tend to move along these planes if the driving force has a nonzero component in any in-plane direction. Such a guided motion of vortices in a PPP leads to the appearance of a $\rho_\perp^+$ contribution to the transverse (with respect to the current direction) magnetoresistivity which is even with respect to the magnetic field reversal.[13,15] It may as well strongly modify the usual odd transverse Hall response.[8,9]

Rather simple formulas were derived recently in Ref. 8 for the experimentally observable nonlinear even and odd longitudinal and transverse magnetoresistivities $\rho^\pm_{\parallel,\perp}(j,\theta,\alpha,\varepsilon)$ as functions of the transport current density j, dimensionless temperature θ, and relative volume fraction $0<\varepsilon<1$ occupied by the parallel twin planes (TP's) directed at an angle α with respect to the current direction. The $\rho^\pm_{\parallel,\perp}$-formulas were presented in Ref. 8 as linear combinations of the even and odd parts of the function $\nu(j,\theta,\alpha,\varepsilon)$ which can be considered as the probability of overcoming the potential barrier of the twins.

From the mathematical viewpoint, the problem, as solved in Refs. 5 and 8 reduces to the Fokker-Planck equation of the one-dimensional vortex dynamics[7] because the vortex motion is unpinned in the direction which is parallel to the TP's. As a consequence, a critical current j_c exists only for the direction which is strictly perpendicular to the TP's ($\alpha=0$); $j_c(\alpha)=0$ for any other direction ($0<\alpha\leqslant\pi/2$). Physically speaking, this is typical for a vortex liquid response.[8] However, the measurements of the response in the solid vortex phase always show that $j_c(\alpha)>0$ for all α (Ref. 15) [although $j_c(\alpha)$ may be anisotropic]. So, in spite of some merits of a model with one set of parallel TP's, which was an exactly solvable stochastic nonlinear model of anisotropic pinning, it cannot describe the j_c anisotropy of the solid vortex phase.[15] The simplest model, which demonstrates this j_c anisotropy for all α with a PPP, is the bianisotropic pinning model with a separable potential formed by the sum of two washboard PPP's in two mutually perpendicular directions (see below).

(a) Main relations. The Langevin equation for a vortex moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}=\mathbf{n}B$ ($B\equiv|\mathbf{B}|,\mathbf{n}=n\,\mathbf{z}$, $\mathbf{z}$ is the unit vector in the z direction, and $n=\pm1$) has the form

$$\eta\mathbf{v}+n\alpha_H\mathbf{v}\times\mathbf{z}=\mathbf{F}_L+\mathbf{F}_p+\mathbf{F}_{th}, \tag{1}$$

where $\mathbf{F}_L=n(\Phi_0/c)\mathbf{j}\times\mathbf{z}$ is the Lorentz force (Φ_0 is the magnetic-flux quantum, c is the speed of light), $\mathbf{F}_p=-\nabla U_p$ is the pinning force (U_p is the pinning potential), $\mathbf{F}_{th}$ is the thermal fluctuation force, η is the vortex viscosity, and α_H is the Hall constant. The fluctuation force $\mathbf{F}_{th}(t)$ is represented by a Gaussian white noise, whose stochastic properties are specified by the relations $\langle F_{th,i}(t)\rangle=0$ and $\langle F_{th,i}(t)F_{th,j}(t')\rangle=2T\eta\delta_{ij}\delta(t-t')$, where T is the temperature in energy units. Our bianisotropic PPP is assumed to be separable and two-periodic, i.e., $U_p(x,y)=U_{pa}(x)+U_{pb}(y)$, $U_{pa}(x)=U_{pa}(x+a)$, $U_{pb}(y)=U_{pb}(y+b)$, where a and b are the periods in x and y direction, respectively. Due to the separability of $U_p(x,y)$, two-dimensional Fokker-Planck equation consistent with (1) can be solved. After some calculations the solution of the problem can be presented as

$$\langle v_x\rangle=\tilde{\nu}_a(F_x)/\eta(1+\epsilon^2),\langle v_y\rangle=\tilde{\nu}_b(F_y)/\eta(1+\epsilon^2). \tag{2}$$

where $\epsilon\equiv\alpha_H/\eta$, $l=a,b$, and F_x,F_y have to be determined from the equations

$$\begin{aligned} F_x&=f_x+\epsilon Z_b[f_y-\epsilon Z_a(F_x)],\\ F_y&=f_y-\epsilon Z_a[f_x+\epsilon Z_b(F_y)]. \end{aligned} \tag{3}$$

Here $f_x\equiv F_{Lx}-\epsilon F_{Ly}$, $f_y\equiv F_{Ly}+\epsilon F_{Lx}$ are known, $Z_l(F)\equiv F-\tilde{\nu}_l(F)$ is the average pinning force (up to a sign) taken in appropriate direction, and

$$1/\tilde{\nu}_l(F) \equiv 1/[F\nu_l(F)] = \{lT[1-\exp(-Fl/T)]\}^{-1} \times \int_0^l dx \exp(-Fx/T) \int_0^l dx' \times \exp\{[U_{pl}(x+x') - U_{pl}(x')]/T\}, \quad (4)$$

where the function $\nu_l(F)$ has the physical meaning of the probability of the vortex overcoming the potential barrier in the x or y direction, respectively, under the influence of an effective force F.[8]

Because the ν- and Z-functions are known for a given pinning potential U_{pl}, the solutions F_x, F_y of Eqs. (3), being substituted in Eqs. (2), give the desired $\langle v_x \rangle$ and $\langle v_y \rangle$. For the case of pinning by only one set of parallel planes $\nu_b = 1$, $Z_b = 0$, $F_y = F_{Ly}(1+\epsilon^2) + \epsilon\tilde{\nu}_a(f_x)$, $F_x = f_x$, and one obtains the results presented in.[8] Below I analyze the solutions of Eqs. (3) in three interesting but physically different cases: $0 < \epsilon \ll 1$, $\epsilon = 0, \epsilon \gg 1$.

For $\epsilon \ll 1$ it follows from Eqs. (3) that $F_y = F_{Ly} + \epsilon\tilde{\nu}_a(F_{Lx}) + O(\epsilon)$, $F_x = F_{Lx} - \epsilon\tilde{\nu}_b(F_{Ly}) + O(\epsilon)$. Then, using Eqs. (2) and the relation $\mathbf{E} = (B/c)(\mathbf{n} \times \langle \mathbf{v} \rangle)$, I obtain the resistivity tensor (in xy representation) as

$$\hat{\rho} = \rho_f \begin{pmatrix} \nu_b(\tilde{F}_y) & -\epsilon\nu_a(F_{Lx})\nu_b(F_{Ly}) \\ \epsilon\nu_a(F_{Lx})\nu_b(F_{Ly}) & \nu_a(\tilde{F}_x) \end{pmatrix}, \quad (5)$$

where $\tilde{F}_x \equiv F_{Lx} - \epsilon\tilde{\nu}_b(F_{Ly})$, $\tilde{F}_y \equiv F_{Ly} + \epsilon\tilde{\nu}_a(F_{Lx})$, $F_{Lx} = n(\Phi_0/c)j_y$, $F_{Ly} = -n(\Phi_0/c)j_x$, and $\rho_f \equiv B\Phi_0/\eta c^2$.

The longitudinal and transverse (relative to the current direction) components of the dimensionless (in units of ρ_f) resistivities are

$$\rho_\parallel^+ = \nu_b(F_{Ly})\sin^2\alpha + \nu_a(F_{Lx})\cos^2\alpha,$$
$$\rho_\perp^+ = [\nu_a(F_{Lx}) - \nu_b(F_{Ly})]\sin\alpha\cos\alpha, \quad (6)$$

$$\rho_\parallel^- = \nu_b^-(\tilde{F}_y)\sin^2\alpha + \nu_a^-(\tilde{F}_x)\cos^2\alpha,$$
$$\rho_\perp^- = \epsilon\nu_a(F_{Lx})\nu_b(F_{Ly}) + [\nu_a^-(\tilde{F}_x) - \nu_b^-(\tilde{F}_y)]\sin\alpha\cos\alpha, \quad (7)$$

where α is the angle between the $\mathbf{j}$ and the $\mathbf{y}$ vector, see Fig. 1, $\pm$ denote even and odd components of the ν_l functions and resistivities with respect to the magnetic field reversal[8] and

$$\nu_b^-(\tilde{F}_y) \approx \epsilon\nu_b'(F_{Ly})\tilde{\nu}_a(F_{Lx}),$$
$$\nu_a^-(\tilde{F}_x) \approx -\epsilon\nu_a'(F_{Lx})\tilde{\nu}_b(F_{Ly}). \quad (8)$$

Eqs. (6)–(8) are accurate to the first order in $\epsilon \ll 1$ and contain a lot of new physical information which will be elaborated on elsewhere.[18] However, it is instructive to discuss in short the main physically important features of Eqs. (6)–(8). As it follows from them, the $\rho_{\|,\perp}^{\pm}$ magnetoresistivities can be found if ν_l-functions are known. The converse statement is also valid: it is possible to reconstruct ν_l-functions from (j,θ) resistive measurements (see b item below). The key point in the physical interpretation of Eqs. (6)–(8) is the treatment of the ν_l-functions $(l = a,b)$ as the (F_l, θ, α)-dependent probabilities of overcoming by vortices the potential barrier in the x or y direction under the influence of the effective forces F_x or F_y, respectively.[8] These ν_l-functions describe an essentially nonlinear transition from the linear low-temperature thermoactivated flux flow (TAFF) regime of vortex motion to the ohmic flux flow (FF) regime and at small temperature θ or current density j the $\nu_l(j,\theta)$ are step-like functions of the j or θ, respectively (see Fig. 4 and 5 in Ref. 8). It follows from Eqs. (6)–(8) that for $\alpha \neq 0, \pi/2$ the observed resistive response contains not only the ordinary longitudinal $\rho_{\|}^+(\alpha)$ and transverse $\rho_\perp^-$ magnetoresistivities, but also two new components induced by the pinning anisotropy: an *even transverse* component $\rho_\perp^+(\alpha)$ and an *odd longitudinal* $\rho_{\|}^-(\alpha)$. The physical origin of the $\rho_\perp^+(\alpha)$ (which is independent of ϵ) is related in an obvious way with the possibility (in TAFF regime) of guided vortex motion along the "channels" of the washboard PPP. On the other hand, the component $\rho_{\|}^-(\alpha)$ is proportional to the odd components ν_l^-, which are zero at $\epsilon = 0$ and have a maximum in the region of the nonlinear transition from the TAFF to the FF regime (see Figs. 6 and 7 in Ref. 8) at $\epsilon \neq 0$. In the (j,θ)-dependence of the odd transverse (Hall) resistivity $\rho_\perp^-(j,\theta)$ there are contributions both from the even $\nu_l^+ \approx \nu_l$ and from the odd ν_l^- components of the $\nu_l(j,\theta)$-functions, whose relative magnitudes are determined by the angle α and the Hall constant ϵ. Note, that as the odd longitudinal $\rho_{\|}^-$ and odd transverse $\rho_\perp^-$ magnetoresistivities arise thanks to the Hall effect, their characteristic scale is proportional to $\epsilon \ll 1$ [see Eqs. (7) and (8)]. Below I elaborate on bianisotropic guiding effects and new scaling relations for the Hall conductivity ϵ.

(b) Guiding analysis $(\epsilon = 0)$. Neglecting the Hall effect, we first consider in Eqs. (6) two special cases where the transport current is directed either along the x axis $(\mathbf{j}\|\mathbf{x}, \alpha = \pi/2, X$ geometry of experiment) or along y axis $(\mathbf{j}\|\mathbf{y}, \alpha = 0, Y$ geometry). In both of these cases only longitudinal resistivities $\rho_\|^x(j,\theta) = \nu_b(j,\theta)$ and $\rho_\|^y(j,\theta) = \nu_a(j,\theta)$ exist and $\rho_\perp^x = \rho_\perp^y = 0$. So, the measurement of longitudinal resistivities in the XY geometries allows us to deduce the current and/or temperature dependence of the $\nu_{b,a}$ functions, which are the main nonlinear components of the presented theory. The knowledge of $\rho_\|^{x,y}(j,\theta)$ is also sufficient for the calculation of $\rho_{\|,\perp}^\alpha(j,\theta)$ at arbitrary angle α because Eqs. (6) can be rewritten as

$$\rho_\|^\alpha(j,\theta) = \rho_\|^x(j_x,\theta)\sin^2\alpha + \rho_\|^y(j_y,\theta)\cos^2\alpha,$$
$$\rho_\perp^\alpha(j,\theta) = \sin\alpha\cos\alpha[\rho_\|^x(j_x,\theta) - \rho_\|^y(j_y,\theta)]. \quad (9)$$

Note the nontrivial nonlinear angular dependence in the arguments of the $\rho_\|^x$ and $\rho_\|^y$ functions. The equations (9) also show how the peculiarities of $\rho_\|^{x,y}(j,\theta)$ in the "basic" XY geometries are manifest in the $\rho_{\|,\perp}^\alpha(j,\theta)$. First I shall study the α dependence of the critical current density $j_c(\alpha,\theta)$ in terms of the basic temperature dependent critical current densities of our model $j_c^x(\theta) \equiv j_c(\pi/2,\theta)$ and $j_c^y(\theta) \equiv j_c(0,\theta)$. At

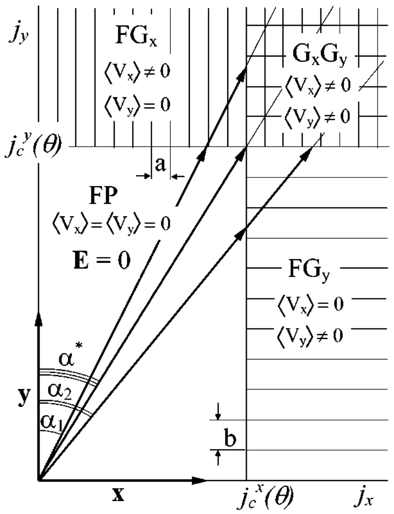

FIG. 1. Schematic diagram of the dynamic states of the vortex system on the (j_x, j_y) plane at $T \geqslant 0$. There are four regions: FP (full pinning), FG_x (full guiding along **x**), FG_y (full guiding along **y**), and G_xG_y (guiding in both the x and y directions). $j_c^y(\theta)$ and $j_c^x(\theta)$ are the temperature-dependent critical current densities in Y and X geometries, and a,b are the periods along the x and y axes, respectively. α is the angle between the **j** and **y** vectors, $\alpha_1 < \alpha^*$, and $\alpha_2 > \alpha^*$.

$\theta \ll 1$ these current densities define a sharp transition from the low-resistivity TAFF-regime (with $\nu_{TAFF}(\theta) \sim \exp(-1/\theta) \ll 1$ and resistivity $\rho_{TAFF} \ll 1$) to the high-resistivity flux flow (FF) regime with $\nu_{FF}(\theta) \approx 1$ and resistivity $\rho_{FF} \approx 1$.[8] In Fig. 1 the first quadrant of the (j_x, j_y) plane is divided by straight lines $j_y = j_c^y(\theta)$ and $j_x = j_c^x(\theta)$ into four regions. The tip of the **j** vector with coordinates $(j\sin\alpha, j\cos\alpha)$ can belong to each of these regions with different physical meanings of the respective dynamic states of the vortex system.

The region of "full pinning" (FP in Fig. 1), where $\rho_{\parallel TAFF}^x \ll 1, \rho_{\parallel TAFF}^y \ll 1$ is shown by the unshaded rectangle and its diagonal determines the critical angle $\alpha^* = \alpha^*(\theta)$ $(\tan\alpha^* \equiv j_c^x/j_c^y)$. Then it is easy to see that

$$j_c(\alpha,\theta) \equiv \begin{cases} j_c^>(\alpha,\theta) = j_c^x(\theta)/\sin\alpha & \alpha > \alpha^* \\ j_c^<(\alpha,\theta) = j_c^y(\theta)/\cos\alpha & \alpha < \alpha^*. \end{cases} \tag{10}$$

In the region of "full X guiding" (FG_x in Fig. 1), shaded by vertical lines, $\alpha < \alpha^*, j_x < j_c^x(\theta)$, and the vortices are moving along the x-axis, i.e., $\langle \mathbf{v}\rangle_{FG_x} \| \mathbf{x}$. The "full Y guiding" ($FG_y$) region is shaded in Fig. 1 by horizontal lines. It represents the fully guided motion of vortices along the y axis with $\langle \mathbf{v}\rangle_{FG_y} \| \mathbf{y}$, because at $\alpha > \alpha^*$ we always have $j_y < j_c^y(\theta)$. And lastly, the "XY guiding" (G_xG_y) region, shaded by crossed lines, characterizes the coexistence of guiding in both directions, where $\langle \mathbf{v}\rangle = \langle v_x\rangle \mathbf{x} + \langle v_y\rangle \mathbf{y}$ with $\langle v_x\rangle \neq 0$ and $\langle v_y\rangle \neq 0$.

If for the given sample with fixed α the transport current is increasing from zero, then, depending on the α, j values it is possible to realize sequentially different variants of intersection by the tip of the **j** vector of the boundaries between the neighboring regions (see Fig. 1). For example, if $\alpha = \alpha_2 > \alpha^*$, then the series of intersections $FP \to FG_x \to G_xG_y$ follow. Since a new source of dissipation appears at each of the intersections, the longitudinal resistivity of the sample $\rho_\parallel^\alpha(j,\theta)$ acquires a kink (inflection point) at the corresponding values of j.[18] In general, there are two such kinks on the $\rho_\parallel^\alpha(j,\theta)$ dependence (if $\alpha \neq \alpha^*, 0, \pi/2$); only in the case $\alpha = \alpha^*$ these two kinks merge into one.

(c) Scaling relations for the Hall conductivity ϵ. From Eqs. (7) new scaling relations can be derived. They are valid for $\epsilon \ll 1$ in the range of α not close to $\alpha = 0, \pi/2$ and allow to express the Hall constant in terms of the experimentally measured resistivities $\rho_{\parallel,\perp}^{\pm}(\alpha, j, \theta)$. Namely, from Eqs. (6) and (7) new scaling relations for ϵ follow in two equivalent forms. This means that it is sufficient to know from measurements only one of the two odd resistivities [compare with Eqs. (40) in Ref. 8]:

$$\epsilon = (\rho_\parallel^- \tan\alpha - \rho_\perp^-)/[\nu_a'(j_x)\tilde{\nu}_a(j_y)\tan\alpha - \nu_a(j_y)\nu_b(j_x)],$$
$$\epsilon = (\rho_\parallel^- + \rho_\perp^- \tan\alpha)/[\nu_a(j_y)\nu_b(j_x)\tan\alpha - \nu_a'(j_y)\tilde{\nu}_b(j_x)]. \tag{11}$$

Here the ν functions are given by inverting Eqs. (6) and I omitted the $\alpha - j - \theta$ variables of the $\rho_{\parallel,\perp}^-$ functions for brevity.

(d) Dominating Hall conductivity $(\epsilon^{-1} \equiv \Delta \to 0)$. This limit may be interesting experimentally for the YBaCO single crystals.[19] On the other hand, the "zero approximation" (for $\Delta = 0$) at arbitrary angle α is not evident. As it will be verified below [see Eqs. (13)], the $\epsilon \to \infty$ limit corresponds to the limit $F \to \infty$ in $\nu_a(F)$ and $\nu_b(F)$. Recalling that the $\nu(F)$ are even steplike functions of F and $\lim_{F\to\infty}\nu(F) = 1$, we conclude that $\nu(F) \approx 1 - (F_p/F)^2$, where F_p has a meaning of the average pinning force. Then ν_a and ν_b can be written (see also Ref. 8) as

$$\nu_a(F) \approx 1 - (F_{pa}/F)^2, \quad \nu_b(F) \approx 1 - (F_{pb}/F)^2, \tag{12}$$

where F_{pa} and F_{pb} are the average pinning forces in the x and y directions, respectively. In this case the definition of $Z(f)$ yields $Z_b(F_y) = F_{pb}^2/F_y, Z_a(F_x) = F_{pa}^2/F_x$. Then it follows that F_x and F_y satisfy two similar quadratic equations and their solutions for $\Delta \to 0$ are

$$F_x = -F_{Ly}/\Delta + F_{Lx} + F_{pb}^2/F_{Lx} + O(\Delta),$$
$$F_y = F_{Lx}/\Delta + F_{Ly} + F_{pa}^2/F_{Ly} + O(\Delta). \tag{13}$$

Next, using Eqs. (2) I conclude that

$$\langle v_x\rangle = -F_{Ly}/æ + \Delta(F_{Lx}/æ)(1 + F_{pb}^2/F_{Lx}^2),$$
$$\langle v_y\rangle = F_{Lx}/æ + \Delta(F_{Ly}/æ)(1 + F_{pa}^2/F_{Ly}^2), \tag{14}$$

where $æ \equiv n\alpha_H \to \pm\infty$ and $\eta = \text{const}$. It follows from Eqs. (14) that the resistivity tensor in xy representation is

$$\hat{\rho}=\rho_{æ}\begin{pmatrix}\Delta(1+F_{pa}^2/F_{Ly}^2) & -1\\ 1 & \Delta(1+F_{pb}^2/F_{Lx}^2)\end{pmatrix}. \quad (15)$$

Here $\rho_{æ}\equiv B\Phi_0/æc^2$ is the "net" Hall resistivity and $\rho_{æ}(\mathbf{H})=-\rho_{æ}(-\mathbf{H})$. As $\Delta\to 0$, we see from Eq. (15) that the diagonal (dissipative) components of the $\hat{\rho}$- tensor are smaller by a factor of Δ in comparison with the nondiagonal (nondissipative) components $\pm|\rho_{æ}|$. These are not explicitly depending on Δ within the main approximation.

Further calculation of $\rho^{\pm}_{\parallel,\perp}(j)$ yields:

$$\rho_{\parallel}^{+}(j)=\Delta\rho_{æ}[1+(j_{cx}^2+j_{cy}^2)/j^2],\quad \rho_{\perp}^{-}=\rho_{æ},$$
$$\rho_{\perp}^{+}(j)=\Delta\rho_{æ}[j_{cy}^2\tan\alpha-j_{cx}^2\cot\alpha]/j^2, \rho_{\parallel}^{-}=0. \quad (16)$$

Here I took into account that $(F_{pl}/F_{Ll})^2=(j_{cl}/j_l)^2$, $l=x,y$. From Eqs. (16) several physical findings follow *a*) all nonzero resistivities tend to go to zero as $\Delta\to 0$ (at $\eta=const$), *b*) $\rho_{\parallel}^{-}=0$ and $\rho_{\parallel}^{+}(j)$ does not depend on the angle α, *c*) $\rho_{\perp}^{+}(j)$ is current, angle and pinning dependent, *d*) $\rho_{\perp}^{-}=\rho_{æ}$ is current, angle and pinning independent, *e*) the ratio $\rho_{\perp}^{+}(j)/\rho_{\parallel}^{+}(j)=-\cot\beta$, which determines the angle between $\mathbf{j}$ and $\langle\mathbf{v}\rangle$ [see Ref. 8, Eq. (36)] is decreasing with j increasing, *f*) there are some limitations on the α values in the vicinity of $\alpha=0,\pi/2$ following from the derivation of the approximate Eqs. (14).

In conclusion, we derived bianisotropic nonlinear resistive properties of the two-dimensional stochastic PPP-model which can be realized experimentally both in naturally grown and artificially prepared pinning structures. For instance, parallel nanocracks, which are normal to the twin planes, were found in unidirectionally twinned *c*-axis films of $YBa_2Cu_3O_{7-\delta}$ on $NdGaO_3$ substrates.[20] Artificial bianisotropic pinning array of narrow magnetic stripes (instead of a regular array of magnetic dots) can be prepared by the same methods which were used in.[21,22] Besides, it seems possible to create different bianisotropic pinning structures which combine both naturally grown systems of parallel plane defects and artificially prepared orthogonal pinning structures with a PPP. For example, in the untwinned *a*-axis oriented YBCO film,[23] which is covered by a magnetic tape with a pre-recorded periodic signal[24] it is feasible to study a bi-bianisotropic competition between the intrinsic pinning of a layered YBCO structure and the artificial programmable magnetic pinning. In contradistinction to the model with one set of PPP's,[5–8] this model predicts a $j_c(\theta)$ anisotropy and relates it to the guiding anisotropy, describing the appearance of two steplike and two bumplike singularities in the $\rho^{+}_{\parallel,\perp}$ and $\rho^{-}_{\parallel,\perp}$ (Hall) resistive responses, respectively. It should be noted that similar results [Eqs. (6)–(7)] may be also obtained within the frame of nonlinear phenomenological approach[4] which takes into account the vortex-vortex interactions, ignored in the presented model. But the stochastic approach allows one to calculate the nonlinear ν_l-functions for a given pinning potential [see Eq. (4)], whereas in the phenomenological approach $\nu_l(B,\Theta,j)$ dependences cannot be calculated and should be determined from the experimental measurement of the longitudinal resistivities in XY geometries.

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